Bond Analysis: The Concept of Duration

Bondholders can be hurt by a number of circumstances: the issuer may decide to redeem the bonds before the maturity date, the issuer may default or interest rates may fluctuate and reduce the overall return of the bonds to the investor.

The first two risks, call risk and default risk, can be minimized by careful bond selection and by constructing a bond portfolio that is adequately diversified. The risk of changing interest rates, however, can be evaluated and minimized by employing the concept of duration.

The concept was developed in 1938. But interest in duration really took off in the mid-1970s, when interest rates became more volatile.

Duration has been defined as a maturity measure that takes into account not only the redemption date, but also the dates on which interest is paid and the amount of interest. It is like an average time the bondholder must wait to be paid, reflecting the amount and timing of every cash flow rather than merely the length of time until the final payment occurs. Specifically, it is the time-weighted present value of all cash flows, divided by the bond price.

The concept is useful to bond investors for a number of reasons:

- It provides a uniform measure of price sensitivity to interest rate changes, valid for all combinations of coupons and maturities, that can be used to anticipate price variability;
- It specifies a holding period, which can substantially reduce exposure to price risk and reinvestment rate risk; and
- It permits investors to construct and maintain bond portfolios that will provide a desired realized return over a specified holding period.

An understanding of how duration works and how it can be applied will greatly expand a bond investor's ability to maximize return and minimize risk.

Measuring Price Sensitivity

One benefit of duration is that it provides a uniform measure for comparing bond price sensitivities for all combinations of coupons and maturities. This is necessary because bond prices fluctuate as interest rates change: When interest rates rise, bond prices decline; when rates fall, bond prices increase. But they do so at different rates, depending on their individual characteristics. Interest rate changes will cause greater price changes in bonds with lower coupons and longer maturities than in bonds with higher coupons and shorter maturities. Duration accounts for these variables and allows investors to compare different types of bonds.

Viewed from another perspective, bonds with markedly different coupons and maturities may have the same duration and, therefore, the same anticipated price variability. For example, examine Bonds A, B and C, all with a $1,000 par value.

	Coupon	**Years to maturity**	**Price**
Bond A	16%	28	$1,200.00
Bond B	7%	23	$500.50
Bond C	10%	15	$902.50

While these bonds exhibit a wide range of characteristics, all have a duration of 7.72 years, and all will react similarly to a change in the level of interest rates. This observation would not have been readily apparent from examining the coupons, maturities and prices alone.

For a given interest rate change, a bond with a duration of 8.0 will experience a percentage price change twice that of a bond with a duration of 4.0. This property of duration permits investors to evaluate bonds with very different characteristics and directly compare their reactions to interest rate fluctuations. Bond speculators, who trade bonds to take advantage of price fluctuations, use this property of duration to enhance profits. If an interest rate decline is predicted, they would buy. They sell short if a rate increase is expected. But either strategy would provide greater profits with long-duration bonds than those with short durations, because the price fluctuation will be greater for the long-duration bonds.

The conservative investor who buys bonds as a long-term investment might adopt exactly the opposite strategy. That is, he may want to buy bonds of shorter duration so that price fluctuations will be minimized. But this strategy only addresses one aspect of bond risk.

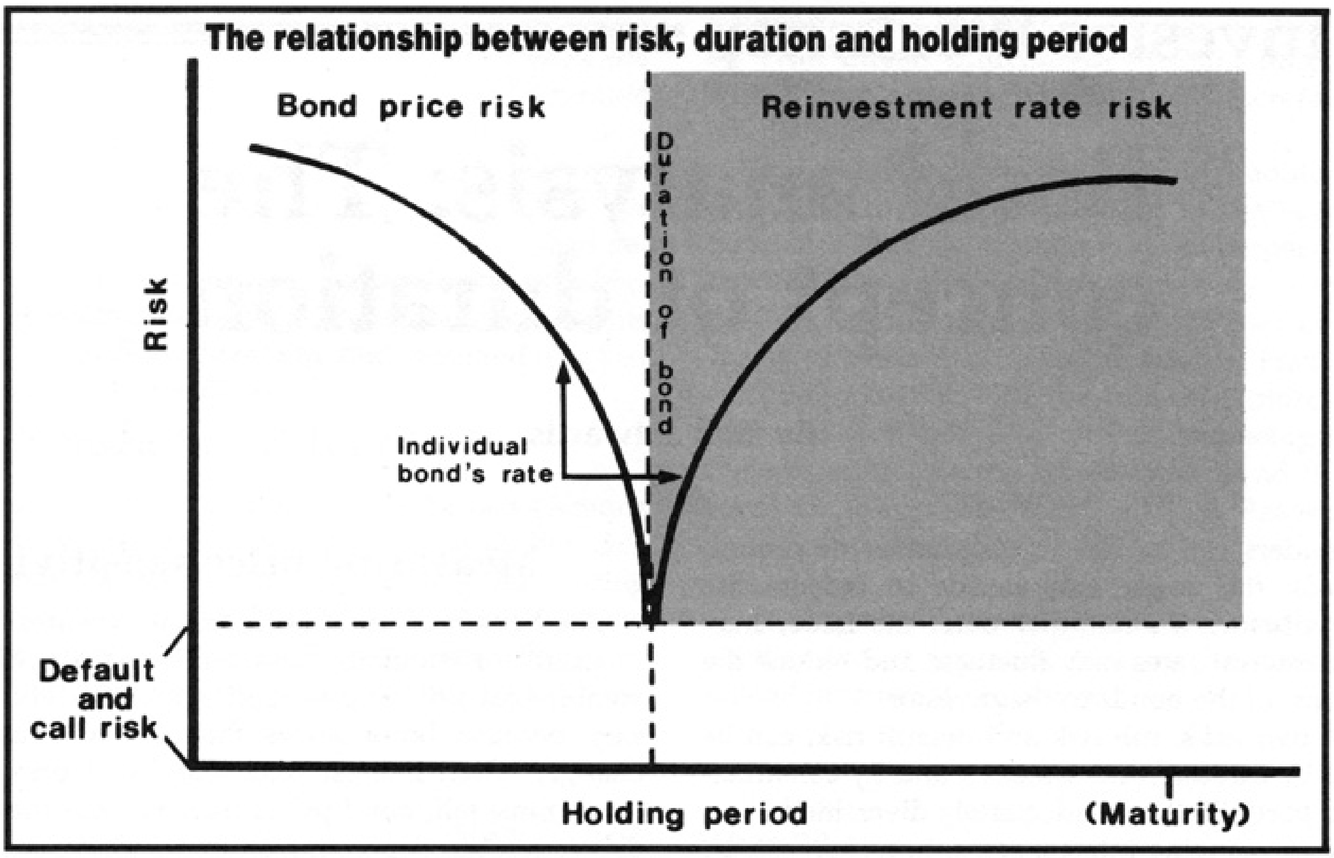
Balancing the Risks

Bondholders face two major interest rate risks: the price risk already mentioned, and the reinvestment rate risk. The latter affects bonds in a manner opposite to the price risk: Bondholders benefit from interest rate increases in terms of the reinvestment rate, and are hurt by rate drops. This is because when interest rates rise, coupon payments can be reinvested at a higher interest rate, which has a positive impact on overall bond yield. When interest rates drop, coupons can be reinvested only at the lower rates, which will have a negative impact on the overall bond yield.

Since interest rate changes have an opposite impact on the two risks, there should be some method of balancing the effect to reduce overall risk: If rates rise, the loss to an investor due to the bond price's drop should be offset by the increase in return he will get from reinvesting his coupon payments; and if rates drop, the decreased return he will get from reinvesting his coupon payments should be offset by an increase in the bond's value.

What is this balance point? The bond's duration.

Figure 1.



When a bond is held to its duration, the price risk and the reinvestment rate risk are at a minimum; when it is held either for a longer or shorter time, one of the risks will be increasingly dominant.

Figure 1 above illustrates the point. The procedure for a single bond is presented in Figure 2 (Figure 3 gives the procedure for portfolio duration). The formula for calculating duration is presented in

Figure 4 at the end of the article.

This ability to offset the two risks is the major benefit of applying the duration concept to bond management, particularly since it can be applied to an entire portfolio of bonds, where the duration can be tailored to an investor's needs.

Immunization: Not Just for Smallpox

Duration allows bondholders to construct portfolios that match cash flows to an investor's needs. The duration used is the time the cash flows are required. Since the duration is also the point at which the two interest rate risks are balanced, the investor's returns are "immunized" against interest rate fluctuations. This process of matching portfolio duration with the date funds will be needed is thus known as "immunization."

For instance, if funds will be needed for a college education nine years from now, the investment portfolio that will be used to fund that education should have a duration—not maturity—of nine years.

Figure 2.

Calculating bond duration: The procedure

Calculation example

The procedure is the same for all bonds. Here, we are using a bond with these characteristics:

Coupon: 10% **Term:** 5 years **Payments:** semi-annual **Par value:** $1,000

Price: $859.50 **Yield-to-maturity*:** 14% annually (7% per period)

① Period	② Cash flow	③ Present value factor @ 7%/period	④ Product of ① x ② x ③
1	50	.9346	46.73
2	50	.8734	87.34
3	50	.8163	122.44
4	50	.7629	152.58
5	50	.7130	178.25
6	50	.6663	199.89
7	50	.6227	217.94
8	50	.5820	232.80
9	50	.5439	244.75
10	1050	.5083	5337.15
			⑤ 6819.87

⑥ Duration $= \frac{6819.87}{859.50} = 7.93$ periods

⑦ Duration $= \frac{7.93}{2} = 3.97$ years

Calculation explanation:

① List periods in which cash flows will be received by number. In this example, the bond has 10 cash flow periods since it is a five-year bond paying interest semi-annually.

② List cash flows for each period: Coupon rate × face value ÷ number of interest payments per year = .10 × $1000/2 = $50. Note the face value is included in the final period cash flow.

③ Determine present value factors (PVF) for each period from tables or by calculating: $\text{PVF} = \frac{1}{(1+r)^n}$ where r is the *per period* discount rate $\left(\frac{\text{annual yield-to-maturity}}{\text{\#payments per year}} = \frac{0.14}{2} = 0.07\right)$ and n is the period number. For example, the present value factor for period 8 is: $\text{PVF} = \frac{1}{(1.07)^8} = \frac{1}{1.7182} = 0.5820$

④ Calculate the product of period times cash flow times PVF for each period.

⑤ Add numbers calculated in step ④.

⑥ Divide total from step ⑤ by the market price of the bond to obtain the duration in terms of periods.

⑦ Divide period duration by number of periods (payments) per year to obtain duration in terms of years.

*Yield to maturity: the total return provided by a bond to its maturity date, taking into account the price paid for the bond, the coupon rate, the maturity, and the bond's face value. Bond yields-to-maturity are available from financial publications such as Standard & Poor's Bond Guide or may be obtained by consulting standard bond yield tables.

Figure 3.

Calculating portfolio duration: The procedure

Definition:

The weighted average of the durations of the bonds in the portfolio, weighted by the bonds' market value as a percentage of the portfolio value

$$D_P = \sum_{m=1}^{k} X_m D_m$$

D_P = portfolio duration
k = number of bonds in portfolio
X_m = proportion of portfolio invested in bond m
D_m = duration of bond m

Calculation example:

The calculation is the same for all portfolios. Here, our portfolio has these characteristics:

	① Dollar values	② Proportion of total portfolio	③ Bond duration	④ Product of ② x ③
Bond A	$ 8,976	0.4488	3.6	1.6157
Bond B	$ 3,762	0.1881	7.1	1.3355
Bond C	$ 7,262	0.3631	5.9	2.1423
Total	$20,000			Total: 5.0935

⑤ Portfolio duration: 5.0935

Calculation explanation:

① List and total dollar values of bond holdings

② Calculate proportion of portfolio invested in each bond: $\frac{\text{Bond Value}}{\text{Portfolio Value}}$

③ List duration of each bond

④ Multiply duration by bond percentage of portfolio, i.e. ② x ③

⑤ Add results of ④ to obtain portfolio duration

The duration—holding period matching principle is straightforward in theory, but implementation can be rather complicated. The major problem arises because each time an interest rate change occurs, the duration changes. So, a duration calculation is good only until a rate change occurs, at which time duration must be recalculated. An interest rate increase will result in a reduced duration, while a rate decline will extend the duration period. Therefore, if one is to keep duration matched with a required holding period, active bond management will be necessary.

It is impossible to alter the duration of a single bond. But a portfolio can be adjusted easily to keep its duration matched to the required holding period. When interest rates increase, short-duration bonds can be sold or long-duration bonds can be added to bring the overall portfolio duration back to its original value. When rates decline, short durations can be added and long durations sold.

Calculating a portfolio duration is simply a matter of weighting the individual bond durations by the bond's relative value as a percent of the portfolio, and adding these figures up to arrive at the portfolio duration (for the procedure, see example at end). When interest rates change, the duration can be recalculated and appropriate adjustments can be made to the portfolio to restore the duration to its required level.

Obviously, the duration adjustment process has some problems. A portfolio cannot be adjusted with every interest rate change—transaction costs would erode returns. As a practical matter, portfolio managers examine their durations periodically and readjust their portfolios when a substantial duration shift seems apparent. This keeps management costs low, yet provides a reasonable measure of risk protection.

ln addition, some duration matches are difficult. A bond's duration will always be shorter than its maturity—much shorter for higher-coupon bonds. In fact, due to the nature of the duration calculations, a bond that is purchased at par has a duration with an upper limit that is equal to the sum of the coupon rate in decimal form (i.e., 10% is 0.10) and the number of payments a year, divided by the product of those two figures:

(rate + number payments) ÷ (rate × number payments)

Figure 4.

Calculating duration: The formula

$$\text{Duration} = \frac{\sum_{t=1}^{n} \frac{t \times C_t}{(1+r)^t}}{\sum_{t=1}^{n} \frac{C_t}{(1+r)^t}}$$

t = time period in which cash flow is received.
n = number of periods (or years for annual payment bond).
C_t = dollar cash flow in period t.
r = per period yield on bond (yield-to-maturity for annual payment bonds).

Definition:

$$\text{Duration} = \frac{\text{time weighted present value of cash flows}}{\text{bond price}}$$

Using duration: A price change example

Calculation:

Percentage price change is approximately:

$$-\text{Duration} \times \left[\frac{r_1 - r_0}{1 + r_0}\right] \times 100$$

r_0 = present yield-to-maturity
r_1 = projected new yield-to-maturity

Example:

Assume the yield to maturity for our 10% coupon five-year semi-annual payment, $1000 face value bond is expected to increase from its present 14% up to 15%. The approximate percentage price change would be −3.48%:

$$\% \Delta P \approx -3.97 \times \left[\frac{0.15-0.14}{1+0.14}\right] \times 100$$

$$\approx -3.97 \times \frac{.01}{1.14} \times 100$$

$$\approx -.0348 \times 100$$

$$\approx -3.48\%$$

The negative sign reflects the inverse relationship between yield changes and prices.

Therefore, the expected new price would be

P_1 = [present price x (% ΔP)/100] + present price
= $859.50 x (−.0348) + 859.50
= $829.59

Duration provides only an approximate price change, valid for small interest rate changes. The actual new price, which can be calculated by other means, is $828.40.

The Models

Although a duration calculation model is presented in Figure 4, one should recognize that duration measures are approximations. Our measure presumes the yield curve (which relates bond yields to their maturities) is flat—that is, yields are the same for all maturities. It also presumes any interest rate change will be uniform for all maturities. While these presumptions clearly do not reflect reality, our duration measure provides results that are not very different from other, more precise measures that correct these presumptions but involve much more complicated calculations.

In any case, evaluations made on the basis of the duration measure illustrated will produce correct comparative decisions in all but extreme cases.

Whether or not bond investors actually apply a duration strategy to their investment portfolios, awareness of the duration concept should alert investors to the fact that all bonds held longer than their duration period will be exposed to reinvestment risks and all bonds held for less than their duration will be exposed to price fluctuation risk.

Furthermore, an appreciation of the duration concept can help investors properly evaluate the impact of the various factors that contribute to bond risk and indicate the necessary strategies for minimizing those risks.

> *Bob Edwards wrote this article for the March 1984 issue of the* AAII Journal. *At the time, Edwards was director of education services at AAII.*